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Global Emission Ceiling versus International Cap and Trade: What is the Most Efficient System when Countries Act Non-cooperatively?

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Jacqueline Morgan^{*} and Fabien Prieur^{}**

Abstract

We model climate negotiations as a two-stage game. In the first stage of the game, players have to agree on a global emission cap (GEC). In the second stage, they non-cooperatively choose either their emission level or their emission quota, depending on whether emission trading is allowed, under the cap that potentially binds them together. A three heterogeneous player quadratic game serves as a base for the analysis. In this framework, when the cap is non-binding, there exists a unique Nash equilibrium. When the emission cap is binding, among all the coupled constraints Nash equilibria, we select a normalized equilibrium by solving a variational inequality, which has a unique solution. In both scenarios – with and without emission trading – we show that there exists a non-empty range of values for which setting a binding cap improves all players' payoff. It also appears that for some values of the cap, all players get a higher payoff under the GEC system alone than under the international cap and trade (ITC) system alone. Thus, the introduction of a GEC outperforms the ITC system both in terms of emission reduction and of payoff gains.

JEL classification: Q28, C72.

Keywords: environmental game, climate change, international cap and trade system, national emission quotas, global emission cap, normalized equilibria, variational and quasi-variational inequalities.

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1 Introduction

Since more than fifteen years, countries have been engaged in a process of reduction of Greenhouse gases (GHG) emissions in order to limit the extent of global warming. The first step to the international control of GHG has been reached with the signature of the Kyoto Protocol (1997). During these negotiations, individual emission reduction targets, or put differently individual emission quotas, have been assigned to 41 developed countries.¹ Moreover, the principle of the international trading of emission permits has been adopted.

In spite of this attempt to solve the climate change problem, it is now clear that countries have not yet achieved full cooperation toward its resolution. Firstly, one of the two biggest GHG emitters, the United States, has not ratified the treaty. The second one, China, was not submitted to emission reduction during the first period of implementation and Russia has ratified the treaty at a later date (in 2004). In addition, participating countries have agreed on emission reduction targets that were far from matching scientific recommendations, because they were not stringent enough. More recently, the Copenhagen (2009) summit's objective was to bring all countries involved in the climate change problem – including the USA, China and developing countries – together around a table and to decide on an international timetable for reducing further GHG emissions during the next decades. But, discussions did not lead to real progress. Actually, many countries were reluctant to accept individually constraining emission quotas. Nevertheless, in order to avoid the worst consequences of global warming, it seems that a common view has emerged about the necessity to limit the rise of temperature to 2 degrees C at the horizon of 2050. In a sense, most countries have then agreed on a global emission reduction effort – or emission cap – but have failed to decide how to split this global burden among countries.

Motivated by this last observation, this paper raises the question of whether a system that would consist in setting a global emission cap and letting countries directly choose their emission levels (under this common constraint) can outperform an international cap and trade system where countries have to decide on national emission quotas. This question is addressed in a static game and it is worth mentioning that the environmental game and the subsequent analysis we develop can also be extended to other environmental issues such as water management and biodiversity protection.

In the same vein as Helm (2003), we assume that national governments behave non-cooperatively under any circumstances. Our game has the following features. In each heterogeneous country (or group of countries), polluting firms use a technology that involves a single input, namely,

¹Targets were defined with respect to the reference year 1990 and had to be reached during the first period of commitment 2008-2012.

emissions. Emissions generate profits at home but aggregate emissions are a public bad since they are damaging to all countries. In this context, two general scenarios are envisioned. First, countries directly choose the amount of emissions (business as usual scenario, BAU). Next, we assume that an international cap and trade system (ICT), such as the one designed during the Kyoto protocol, is implemented. This system consists of the choice, by each country, of a national emission quota and of the possibility to trade emission permits on a competitive market.

Comparing these two scenarios, Helm (2003) finds ambiguous results both in terms of aggregate emissions and payoffs. He notably shows that the ICT system may fail to lower aggregate GHG emissions (with respect to the BAU scenario). Emission trading creates a new strategic interaction between players that channels through the market. This may translate into an amount of aggregate emissions higher than emissions released without trading. In this situation, emissions trading may be unanimously preferred by countries because it allows for efficiency gains. In the opposite case, when the ICT succeeds in lowering emissions, some countries may lose from emission trading and consequently may not approve this system. This paper adds to this analysis the opportunity for countries – from now on players – to adopt a global emission cap (GEC) that puts a (potentially binding) ceiling on aggregate emissions. Thus we will have two different regimes in each scenario depending on whether the global cap is binding. Countries can choose between the two scenarios and between the two regimes.

When the emission cap is binding, there exists in general an infinity of social Nash equilibria (Debreu, 1952, also called coupled constraints Nash equilibria) which are the solutions of a quasi-variational inequality (see Baiocchi and Capelo (1984) and, for example, Morgan and Romaniello (2003) and references therein). We therefore resort to the concept of normalized equilibrium introduced by Rosen (1965) and proceed to the selection of a particular equilibrium.

For applications of this equilibrium concept to environmental and economic games, see, e.g., Haurie and Zaccour (1995), Haurie and Krawczyk (1997), Krawczyk (2000, 2005), Krawczyk and Uryasev (2000) and Tidball and Zaccour (2008). Our framework is very close to the pollution control game developed by Tidball and Zaccour (2005), except that in their model each player faces an exogenous environmental constraint and they do not consider emission trading. Actually, their purpose is to compare three scenarios: the Nash equilibrium, the normalized equilibrium and the cooperative solution. The present work also shares similarities with Drouet et al. (2011) who introduce a global emission cap in a game of climate negotiations. Our approach is however different because, in their dynamic framework with an exogenous GEC, they use a numerical approach by implementing an algorithm while we use an analytical approach to compare the different scenarios for a range of values of the cap.

To start with, a general analysis of the normalized equilibrium properties is conducted. We

are in particular emphasizing the conditions under which this equilibrium exists, is unique and displays strategies that are continuous with respect to the global emission cap.

Next, we consider a quadratic game with three players. Players differ with respect to both their environmental concern and their technology. Considering at least three groups of countries allows us to capture the heterogeneity of countries that participate to international climate negotiations. The first player, who is the most aware about environmental issues and who owns the most efficient technology, can be identified as a group of industrialized countries (such as the European Union). The second player represents a group of developing countries. It is also interesting to introduce a third group, say United States, who also owns an efficient technology but has low environmental concern. The first part of the analysis is devoted to the calculation of non cooperative solutions in all scenarios and regimes. When the ceiling is non-binding, the standard case analyzed by Helm (2003), there exists a unique Nash equilibrium. In the second regime with a binding cap, we characterize, for any level of the emission cap, the unique normalized equilibrium and individual payoffs can be defined as functions of this cap.

The last part of the paper addresses the issue of the performance, in terms of emission reduction and payoffs, of the international control of emissions. This issue may be broken down into two different but related questions: What is the impact of the introduction of a binding cap on payoffs? What is the most efficient system when countries act non-cooperatively?

Our results can be summarized as follows. Consider this opportunity to move away from the business-as-usual scenario where countries directly choose their emission levels. Then, two different ways can be followed. On the one hand, countries may adopt a cap and trade system. In our setting, allowing emission trading has no impact on the amount of aggregate emissions released in the atmosphere. The aggregate payoff is higher but one player is always worse off under the ITC system. Thus, there cannot be an unanimous agreement for the implementation of the trading system. On the other hand, the opportunity exists to set a global emission ceiling. This is obviously a means to reduce the amount of aggregate emissions, with respect to the business-as-usual. The comparison between the two regimes also reveals that for some values of the cap, all countries gain from the introduction of a binding cap. Finally, our last result provides strong support for the GEC system alone because we show that some values of the cap exist for which it outperforms the ITC system alone not only in terms of emission reduction but also in terms of individual payoffs. The GEC system alone can thus be unanimously approved when the ITC system cannot.

It is worth mentioning that the GEC has the advantage of circumventing the critical question of how the initial allocation of permits between countries should be determined (this difficulty explaining why international negotiations fail to achieve an efficient agreement) because countries

only need to agree with each other on the global emission ceiling to be imposed. They do not have to engage in binding individual quota and can choose freely their emission level under the constraint imposed (but accepted) by the ceiling. This clearly echoes what has been observed in Copenhagen.

The paper is organized as follows. Section 2 presents the model, defines the solution concept used for the analysis and investigates properties of existence and uniqueness of the normalized equilibrium. Section 3 considers a quadratic game and provides a characterization of the equilibrium, in all scenarios. In section 4, a comparison between scenarios and regimes is conducted, particular attention being paid to players' payoffs in each possible situation. Section 5 concludes.

2 The Model

To model climate change negotiations, a two-stage game is developed. In stage 1, players agree on a global emission cap.² In stage 2, two scenarios are envisioned depending on whether the trading of emissions is allowed. Either players choose non-cooperatively their emission levels under the constraint set by the cap. Or, still submitted to the constraint, they decide on their emission allowances and firms can trade emissions permits on an international market.

Let N be the set of players, each indexed by $i = 1, \dots, n$. Emissions of player i , that are denoted by $e_i \geq 0$, are a by-product of production, with a one-to-one relationship. Player i 's individual payoff is the sum of two components: a benefit from individual emissions, $\pi_i(e_i)$ and a "subjective" damage from aggregate emissions, $\nu_i(e)$ with $e = \sum_{i=1}^n e_i$. The latter function reflects more than a real environmental damage incurred by a country. It also encompasses an environmental concern, or awareness, dimension. Actually, it is clear that consequences of climate change will mainly be a matter for future generations. So, what countries reveal in actual climate negotiations is where they have put the balance between economic and environmental targets. By convention, in the remainder of the analysis, for any e , the player with the highest $\nu_i(e)$ will be identified as the one who cares the most about the environment.

We define as $\alpha > 0$ the global emission cap all players agree on. This GEC is a coupled constraint that binds all players together:

$$e = \sum_{i=1}^n e_i \leq \alpha \tag{1}$$

²The minimum requirement for this kind of agreement to emerge is that individual's payoffs under the cap are higher than what they can obtain in the business-as-usual. This will be the purpose of section 5 to determine under which conditions this can arise.

Denote the vector of emissions by $\bar{e} = (e_1, \dots, e_i, \dots, e_n)$ and the constraint set by:

$$\mathcal{E}(\alpha) = \{\bar{e} = (e_1, \dots, e_n) \in \mathbf{R}_+^n / e = \sum_{i=1}^n e_i \leq \alpha\}, \quad (2)$$

Finally denote

$$\bar{e}_{-i} = (e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n), \quad \bar{e} = (e_i, \bar{e}_{-i}) \text{ and } e_{-i} = \sum_{j \neq i, j \in N} e_j.$$

2.1 First scenario: without emission trading

In the first scenario, players directly choose their emission level under the constraint $\bar{e} \in \mathcal{E}(\alpha)$. Player i 's individual payoff is simply given by:

$$W_i(\bar{e}) = \pi_i(e_i) - \nu_i(e). \quad (3)$$

Since the GEC implies a joint (coupled) constraint on the strategy spaces of all players, one has to look at social Nash equilibria (Debreu, 1952).

We will say that a vector $\bar{e}(\alpha) \in \mathcal{E}(\alpha)$ is a social Nash equilibrium of the pseudogame $\Gamma(\alpha) = \{(W_i)_{i \in N}, \mathcal{E}(\alpha)\}$, associated with the payoff $W_i, i \in N$, and with the set $\mathcal{E}(\alpha)$, if:

$$\text{For all } i \in N, W_i(\bar{e}(\alpha)) \geq W_i(e_i, \bar{e}_{-i}(\alpha)) \text{ for all } e_i \in \mathbf{R} \text{ such that } (e_i, \bar{e}_{-i}(\alpha)) \in \mathcal{E}(\alpha) \quad (4)$$

Under some conditions, solving (4) is equivalent to solving a quasi-variational inequality (see Baiocchi and Capelo (1984) and, for example, Morgan and Romaniello (2003)) that has, in general, an infinite number of solutions. Selecting a normalized equilibrium, as introduced by Rosen (1965), we will have only to solve a variational inequality having a unique solution under appropriate conditions which will be satisfied by the class of quadratic games considered in Section 3. Rosen's approach relies on the definition of a joint payoff function $W(\bar{r}, \bar{e}) = \sum_{i=1}^n r_i W_i(\bar{e})$. The vector $\bar{r} = (r_1, \dots, r_n)$ yields the weights attributed to all countries by a legislator (see Krawczyk, 2005, for a detailed interpretation of these weights). In the subsequent analysis, we will put all these weights equal to one, which implies that the joint payoff function simply corresponds to the aggregate payoff. It can be seen as a natural candidate for being the objective to follow in stage 1 of the game because it boils down to considering that all players involved in the negotiations are treated equally.

Definition 1 A vector $\bar{e}^R(\alpha) \in \mathcal{E}(\alpha)$ is a selected normalized equilibrium (in short: normalized equilibrium) of the pseudogame $\Gamma(\alpha) = \{(W_i)_{i \in N}, \mathcal{E}(\alpha)\}$ if:

$$\sum_{i=1}^n W_i(\bar{e}^R(\alpha)) \geq \sum_{i=1}^n W_i(e_i, \bar{e}_{-i}^R(\alpha)) \text{ for all } \bar{e} \in \mathcal{E}(\alpha). \quad (5)$$

The equilibrium concept of interest being defined, a series of questions naturally arise regarding:

1. The existence of a normalized equilibrium for the pseudogame $\Gamma(\alpha)$;
2. The uniqueness of the normalized equilibrium;
3. The continuity, with respect to α , of the vector equilibrium and of the payoffs at the equilibrium.

The following results can be stated³

Theorem 1 *If π_i and ν_i are continuous function on \mathbf{R}_+ and, for all $\bar{u} \in \mathbf{R}_+^n$, the function $F_{\bar{u}}$ defined by $F_{\bar{u}}(\bar{e}) = \sum_{i=1}^n \pi_i(e_i) - \nu_i(e_i + u_{-i})$ is quasiconcave on \mathbf{R}_+^n , then, for all $\alpha > 0$, there exists at least a normalized equilibrium of the pseudogame $\Gamma(\alpha)$.*

Proof. See the appendix A.

■

In addition, we obtain that

Corollary 1 *If, for all $i \in N$, π_i is concave and ν_i is convex, then, for all $\alpha > 0$, there exists a normalized Nash equilibrium of the pseudogame $\Gamma(\alpha)$.*

Regarding the issue of uniqueness, we use the equivalence between the normalized equilibrium and the solution of a variational inequality. Indeed, a vector $\bar{e}^R(\alpha)$ is a selected normalized equilibrium of the pseudogame $\Gamma(\alpha)$ if and only if it solves the following variational inequality:

$$\langle G(\bar{e}^R(\alpha)), \bar{e}^R(\alpha) - \bar{e} \rangle \geq 0, \text{ for all } \bar{e} \in \mathcal{E}(\alpha) \quad (6)$$

where $G(\cdot)$ is defined, for all $\bar{e} \in \mathcal{E}(\alpha)$, by:

$$G(\bar{e}) = \left\{ \begin{array}{c} \partial_{e_1} W_1(\bar{e}) \\ \cdot \\ \cdot \\ \cdot \\ \partial_{e_n} W_n(\bar{e}) \end{array} \right\}$$

³Proofs, when too long, are relegated to the appendix.

We know, from Rosen (1965), that the normalized equilibria defined above is unique if the operator $-G$ is strictly monotone that is

$$\langle G(\bar{e}) - G(\bar{f}), \bar{e} - \bar{f} \rangle < 0 \quad \text{for all } \bar{e}, \bar{f} \in \mathbf{R}^n.$$

Note that in the framework presented in section 2.3 and analyzed in the subsequent sections, the operator $-G$ is strictly monotone so the selected normalized equilibria defined by condition (5) will be unique.

Assume that there exists a unique normalized equilibrium denoted $\bar{e}^R(\alpha)$.

Theorem 2 *If the functions π_i and ν_i are continuous on \mathbf{R}_+ then $\bar{e}^R(\alpha)$ depends continuously on α on \mathbf{R}_+ .*

Proof. See the appendix B. ■

Finally, we obviously have:

Theorem 3 *If the functions π_i and ν_i are continuous on \mathbf{R}_+ then $W_i^R(\alpha) = W_i(\bar{e}^R(\alpha))$ depends continuously on α on \mathbf{R}_+ .*

2.2 Second scenario: with emission trading

In the second scenario, there exists a market for the trading of emission permits. Players now choose their emission allowance, $\omega_i \geq 0$, still under the coupled constraint $\omega = \sum_{i=1}^n \omega_i \leq \alpha$. This allowance represents the amount of permits given to the representative firm. Representative firms of each country are competitive and can trade permits with each other. In this second scenario, there is a third stage where the representative firm, in each country, chooses the emission level that maximizes profits, taking the market price p and the allowance of permits as given:

$$\max_{e_i} \pi_i(e_i) + p(\omega_i - e_i)$$

Assuming that π_i is sufficiently smooth for all i , then solutions of this problem $e_i(p)$ satisfy $\pi'_i(e_i) = p$ for all i : firms equalize the marginal benefit from emission to the marginal cost, which is simply given by the price. These solutions, together with $\bar{e}(p)$ are uniquely defined for any market price p and continuous with respect to p . Using the market clearing condition, $e = \omega$, we obtain the unique equilibrium price: $p = p(\omega)$. Replacing p by this expression in the one of the emission level, the latter can be expressed as a function of the aggregate allowance $e_i = e_i(\omega)$ and both the equilibrium price and emission levels are continuous with respect to ω . Finally,

the player's individual payoff in this second scenario can be written in terms of the allowance strategies

$$V_i(\bar{\omega}) = \pi_i(e_i(\omega)) + p(\omega)(\omega_i - e_i(\omega)) - \nu_i(\omega),$$

which means that players, or governments, cares about how the market equilibrium changes in response to their strategies.

Therefore, the individual payoff of player i , $V_i(\bar{\omega})$, is a continuous function of $\bar{\omega}$, which means that our analysis of the normalized equilibrium properties (existence, uniqueness and continuity of the normalized equilibrium and of the aggregate value with respect to α), following the introduction of a GEC, can be extended to this second scenario.

The next section introduces a simple three-player quadratic game that will serve as a basis for our analysis.⁴

2.3 The three-player quadratic game

For both scenarios, the two possible problems, depending on whether the emission cap is binding, are addressed. When the emission cap is non-binding, the strategy spaces of players are independent and we are solving for the Nash equilibrium of the game. In the opposite case (the GEC is binding), we turn to the concept of normalized equilibrium.

Within the quadratic game, benefits express as a function of the emission level: $\pi_i(e_i) = e_i(\kappa_i - e_i)$. This is a typical representation of the profit functional, with $\kappa_i > 0$ the exogenous market price of the commodity and a convex production cost. Each player involved in the negotiation process also is characterized by a damage that depends on aggregate emissions: $\nu_i(e) = \frac{\gamma_i}{2}(e)^2$ with $e = \sum_{i=1}^n e_i$ and $\gamma_i \geq 0$. Thus,

$$W_i(\bar{e}) = e_i(\kappa_i - e_i) - \frac{\gamma_i}{2}(e)^2 \quad (7)$$

In our leading example, we introduce a rough distinction between players in order to account for the different groups of countries involved in climate negotiations. This distinction is done with respect to the technology the players own and to their environmental concern:

- Player 1 represents high productivity - high concern countries (for example, Europe),
- Player 2 is a group of high productivity - low concern countries (for example, USA and Russia),

⁴Note that our analysis can be extended to a larger number of countries but, for tractability purposes, we concentrate on this situation.

- Player 3 corresponds to low productivity - low concern countries (developing countries).

This boils down to assuming that:

$$\kappa_1 = \kappa_2 > \kappa_3 \text{ and } \gamma_1 > \gamma_2 = \gamma_3. \quad (8)$$

3 Non-binding GEC: Nash equilibrium

Let us start with the analysis of the standard case where players are not submitted to the coupled constraint. This case has been extensively studied by Helm (2003). Thus, in this section, we merely report the solutions for our particular example and briefly discuss the impact of emission trading on both emissions levels and payoffs. This benchmark is a prerequisite of the original part of our analysis, which will extend the study to the case where a binding GEC is introduced.

3.1 Choice of emission levels

First, we consider the situation where players directly choose their emission levels. At a Nash equilibrium \bar{e}^N , each player solves

$$\max_{e_i} W_i(e_i, e_{-i}^N) = e_i(\kappa_i - e_i) - \frac{\gamma_i}{2}(e_i + e_{-i}^N)^2$$

The unique solution (expressions are symmetric for players $-i$) reads:

$$e_i^N = \frac{\kappa_i(2 + \gamma_{-i}) - \gamma_i \kappa_{-i}}{2(2 + \gamma_i + \gamma_{-i})} \quad (9)$$

Non-negativity of emissions, $e_i^N \geq 0$, can be rewritten as:

$$\frac{\kappa_i}{\kappa_{-i}} \geq \frac{\gamma_i}{2 + \gamma_{-i}} \text{ for } i = 1, 2, 3 \quad (10)$$

these conditions are necessary and sufficient for the existence of a unique non-negative Nash equilibrium. At the Nash equilibrium, each player equalizes the marginal benefit from emissions to the marginal damage. This implies that the second player chooses a level of emissions which is higher than the one chosen by player 1 (that is more concerned with the environment) and by player 3 (that owns a less efficient technology). From now on, we assume that

$$\gamma_1 < 1 + \gamma_3, \quad (11)$$

which is sufficient for having $e_1^N > 0$ and thus $e_2^N > 0$. Assumption (11) states that environmental concerns do not differ too much between countries. It will, in addition, greatly simplify the analysis of the normalized equilibrium.

3.2 Choice of emission allowances

In the situation where emission trading is allowed between firms, players, say governments, have now to choose the emission allowance ω_i that is given for free to firms. Players take into account the impact of their choice on the equilibrium of the market of emission permits. In this second scenario, the representative firm, in each country, chooses the emission level that maximizes profits, taking the market price p and the allowance of permits as given:

$$\max_{e_i} e_i(\kappa_i - e_i) + p(\omega_i - e_i)$$

At this stage, we do not incorporate the non-negativity constraint on e_i . We will a posteriori set the condition such that emissions are non-negative. Thus, one obtains:

$$e_i = \frac{\kappa_i - p}{2} \text{ for } i = 1, 2, 3. \quad (12)$$

From the market clearing condition, $e = \omega$, the equilibrium price

$$p^*(\omega) = \frac{1}{3} (\kappa_i + \kappa_{-i} - 2\omega).$$

By substituting this expression in (12), emission levels can be expressed as functions of the aggregate allowance

$$e_i^*(\omega) = \frac{1}{6} (2\kappa_i - \kappa_{-i} + 2\omega) \quad (13)$$

In our leading example, emission levels are positive for players 1 and 2 (the ones who own the most productive technology, the only thing that matters in determining firms' choices) whereas non-negative emissions for player 3 imposes $\omega \geq \kappa_1 - \kappa_3$. Individual payoffs are defined in terms of emission allowances:

$$V_i(\bar{\omega}) = \frac{1}{36} (2\kappa_i - \kappa_{-i} + 2\omega) (4\kappa_i + \kappa_{-i} - 2\omega) + \frac{1}{18} (\kappa_i + \kappa_{-i} - 2\omega) (4\omega_i - 2\omega_{-i} - 2\kappa_i + \kappa_{-i}) - \frac{\gamma_i}{2} \omega^2 \quad (14)$$

At the Nash equilibrium, player i chooses ω_i that maximizes (14) given ω_{-i}^N . One obtains the unique Nash equilibrium in allowance strategies:

$$\omega_i^N = \frac{\kappa_i(6 + 5\gamma_{-i} - 4\gamma_i) + \kappa_{-i}(2\gamma_{-i} - 7\gamma_i)}{6(2 + \gamma_i + \gamma_{-i})} \quad (15)$$

The non-negativity condition, $\omega_i^N \geq 0$, can be rewritten as:

$$\kappa_i(6 + 5\gamma_{-i} - 4\gamma_i) \geq \kappa_{-i}(7\gamma_i - 2\gamma_{-i}) \text{ for } i = 1, 2, 3, \quad (16)$$

this condition is satisfied for player 2, it is supposed to hold for players 1 and 3. We finally have to check that the emission level of player 3, obtained by replacing ω with ω^N in (13), is non-negative. Emissions of player 3 are non-negative if and only if:

$$\omega^N \geq \kappa_1 - \kappa_3 \Leftrightarrow \frac{\kappa_3}{\kappa_1} \geq \frac{\gamma_1 + 2\gamma_3}{3 + \gamma_1 + 2\gamma_3} \quad (17)$$

The next section proceeds to the comparison of the Nash equilibria obtained in the two scenarios. Particular emphasis is placed on the impact of emission trading on emission strategies, aggregate emissions and payoffs.

3.3 Impact of emission trading on emission levels and payoffs

The comparison between emission and allowance strategies and the position of any player on the market are entirely determined by the damage component of payoffs.

Proposition 1 *In the three player quadratic game,*

i/ The high concern player is a permit buyer and low concern players are permit sellers.

ii/ The high concern player chooses an emission allowance that is lower than the emission level chosen in the absence of trading whereas low concern players' allowances are higher.

Proof. Follows directly from the comparison between i/ (13) and (15) and ii/ (9) and (15). Indeed, $\omega_i^N < e_i^*(\omega^N) \Leftrightarrow \gamma_{-i} - 2\gamma_i < 0$ and $\omega_i^N < e_i^N \Leftrightarrow \gamma_{-i} - 2\gamma_i < 0$. The same inequality is involved in the two comparisons. Under assumption (8), this inequality is satisfied for player 1 whereas the converse holds for players 2 and 3. ■

The interpretation of this result can be found in Helm (2003). To sum up, if a low concern player were to increase its emission allowance, respectively its emission level, its marginal benefit would decrease less rapidly with emission trading. The marginal benefit with trading is given by the permit price $p^*(\omega)$ whereas it is simply given by $\pi'_i(e_i)$ in the absence of trading and one can easily check that $p^*(\omega) = -\frac{2}{3} > \pi''_i(e_i) = -1$. This is due to the fact that an additional unit of ω_i increases the demand of permits $e_i^*(\omega)$ but in a proportion which is less than one because part of this increase is picked up by other players. This is an incentive, for a low concern player, to increase its emission allowance with respect to its emission level. A symmetric reasoning applies to the high concern player.

Regarding the comparison between aggregate emissions, direct calculations yield

$$e^N = \omega^N = \frac{\kappa_i + \kappa_{-i}}{2 + \gamma_i + \gamma_{-i}}, \quad (18)$$

which means that the sum of allowances chosen at the Nash equilibrium, which also yields the aggregate emission level, is the same as the sum of emissions at the Nash equilibrium without emission trading. This is consistent with Helm (2003) who defines the hypothetical scenario where $\omega_i = e_i^N$ and shows that the difference between aggregate emissions is determined by the difference between aggregate marginal benefits obtained in the Nash equilibrium and in the hypothetical scenario: $\omega^N - e^N \lesseqgtr 0 \Leftrightarrow \sum (\pi'_i(e_i^*(e^N)) - \pi'_i(e_i^N)) \lesseqgtr 0$. In our quadratic game, in which second order technological effects are absent ($\pi''(e_i) = -1$ for all i), the latter difference is nil. This means that emission trading has no impact of the amount of emissions released in the atmosphere.

If emission trading does not affect emissions, one obviously expects that it translates into efficiency gains.

Proposition 2 *Emission trading increases the aggregate payoff. As far as individual payoffs are concerned, the high (respectively low) concern player(s)'s payoff is lower (respectively higher) with emission trading.*

Proof. One obtains the Nash equilibrium payoffs in the two scenarios, W_i^N and V_i^N , by substituting respectively the solutions (9) and (15) in (7) and (14). Then, one can verify that $W_i^N < V_i^N \Leftrightarrow -\alpha^2(\gamma_{-i} - 2\gamma_i)^2 < 4\alpha(\gamma_{-i} - 2\gamma_i)(\kappa_i + \kappa_{-i} - 2\alpha)$. Under assumption (8), this inequality is satisfied for players 2 and 3 but not for player 1. In addition, $W^N = \sum W_i^N < V^N = \sum V_i^N$ always holds. ■

The international cap and trade system thus outperforms the regime where players directly choose their emission levels from an aggregate perspective. However, the high concern country loses from the trading of emission permits, which clearly poses the question of the acceptability of such a system. Indeed, since one player is worse off under emission trading, one expects, in the absence of other mechanisms like international transfers, that no unanimous agreement will emerge from any negotiations intended to promote this system (see Helm, 2003).

The purpose of the next sections is to investigate whether the simpler system consisting of choosing a binding cap may be i/ better, in terms of both emissions and payoffs, than the international cap and trade and thus ii/ approved by all the countries involved in climate negotiations.

Let us denote the amount of aggregate emissions released in each scenario (18) by $\bar{\alpha}$. As long as the global emission cap is set to a level $\alpha \geq \bar{\alpha}$, this constraint is not binding that is, the solutions above prevails. But, once this cap is fixed to a level strictly below $\bar{\alpha}$, it plays a role by forcing countries, when choosing e_i or ω_i , to respect it. When the coupled constraint binds players together, we turn to the analysis of the normalized equilibrium.

4 Binding GEC: Normalized equilibrium

4.1 Choice of emission levels

The normalized equilibrium, introduced by Rosen (1965), is indexed by R . Assume that $e^R = \alpha$, $e_i^R \geq 0$ for all i . For the three-player quadratic game, the variational inequality (6) simplifies to:

$$\sum_{i=1}^3 (\kappa_i - 2e_i^R - \gamma_i(e_i^R + e_{-i}^R))(e_i^R - e_i) \geq 0 \quad (19)$$

for all \bar{e} such that $e_i + e_{-i} \leq \alpha$ and $e_i, e_{-i} \geq 0$.

Consider \bar{e} such that $e_i + e_{-i} = \alpha \Leftrightarrow e_{-i}^R - e_{-i} = -(e_i^R - e_i)$ and use this relation to remove e_2 and e_2^R from the variational inequality, then:⁵

$$\sum_{i \neq 2} (\kappa_i - \kappa_2 + (2 + \gamma_2 - \gamma_i)\alpha - 4e_i^R - 2e_{-i \setminus 2}^R)(e_i^R - e_i) \geq 0 \quad (20)$$

for all \bar{e} such that $e_i + e_{-i} = \alpha$ and $e_i, e_{-i} \geq 0$

If the equation $\kappa_i - \kappa_2 + (2 + \gamma_2 - \gamma_i)\alpha - 4e_i^R - 2e_{-i \setminus 2}^R = 0$ holds for $i = 1, 3$, then using the feature that $e_2^R = \alpha - e_{-2}^R$, one gets the interior solution

$$e_i^R(\alpha) = e_i(\alpha) = \frac{2\kappa_i - \kappa_{-i} + \alpha(2(1 - \gamma_i) + \gamma_{-i})}{6} \text{ for all } i = 1, 2, 3 \quad (21)$$

Otherwise, it is possible to obtain corner solutions with $e_i^R(\alpha) = 0$ or $e_i^R(\alpha) = \alpha$. Note that, by definition, $e_2(\alpha) \in [0, \alpha]$ when $e_1(\alpha)$ and $e_3(\alpha)$ also lie in the interval $[0, \alpha]$. From assumption (11), one can easily check that $e_1(\alpha) \geq 0$ and $e_3(\alpha) \leq \alpha$ for all $\alpha \in (0, \bar{\alpha}]$. Define two boundaries $\hat{\alpha}$ and $\tilde{\alpha}$ as:

$$\hat{\alpha} = \frac{\kappa_3 - \kappa_1}{2(-2 + \gamma_3 - \gamma_1)} \quad (\Leftrightarrow e_1(\alpha) = \alpha) \text{ and } \tilde{\alpha} = \frac{2(\kappa_1 - \kappa_3)}{2 + \gamma_1 - \gamma_3} \quad (\Leftrightarrow e_3(\alpha) = 0),$$

these boundaries satisfy: $0 < \hat{\alpha}^w < \tilde{\alpha}^w < \bar{\alpha}$. The normalized equilibrium strategies are given by

$$\left\{ \begin{array}{ccc} (0, \hat{\alpha}) & (\hat{\alpha}, \tilde{\alpha}) & (\tilde{\alpha}, \bar{\alpha}] \\ e_1^R(\alpha) = \alpha & e_1^R(\alpha) = e_1(\alpha) & e_1^R(\alpha) = e_1(\alpha) \\ e_2^R(\alpha) = 0 & e_2^R(\alpha) = \alpha - e_1(\alpha) & e_2^R(\alpha) = e_2(\alpha) \\ e_3^R(\alpha) = 0 & e_3^R(\alpha) = 0 & e_3^R(\alpha) = e_3(\alpha) \end{array} \right\} \quad (22)$$

In our example, assuming (11) is satisfied, one observes that for a low enough α , the high concern - high productivity player is the only one with positive emissions. Actually, it rejects the total amount of allowed emissions. The second intermediate player starts emitting for higher

⁵Denoting by $e_{-i \setminus 2}^R$ the strategy of player other than i and 2.

values of the GEC, its emissions being given by the difference between α and the emissions of player 1. Finally, when the GEC is set to a sufficiently high level, the three players have a positive level of emission given by the interior solution $e_i(\alpha)$.

In addition, focusing on the interval $(\tilde{\alpha}, \bar{\alpha}]$ such that all players release positive emissions, the impact of the introduction of the cap on emission levels is crystal-clear. When submitted to a binding GEC, all players reduce their emission levels: $e_i^R(\alpha) < e_i^N$ for all $\alpha \in (\tilde{\alpha}, \bar{\alpha})$ and for $i = 1, 2, 3$. In addition, emissions are continuous at the critical level $\bar{\alpha}$: $e_i^R(\bar{\alpha}) = e_i^N$ for $i = 1, 2, 3$.⁶ Looking at emission levels under the binding regime also reveals that player 2 still chooses the highest amount of emissions and the ranking between emissions of player 1 and player 3 remains ambiguous. As an illustration of how emission levels change in response to changes in α , see figure 1.⁷

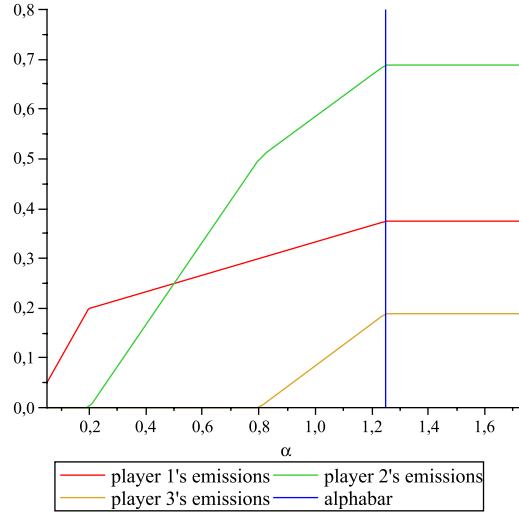


Figure 1: Emission levels, with and without a binding cap.

4.2 Choice of emission allowances

When the emission cap is chosen below $\bar{\alpha}$, the solution (15) is no longer valid and we have to compute the normalized equilibrium. Note that the relevant domain of variation of α is now

⁶Note that emissions are also continuous at the critical boundaries $\hat{\alpha}$ and $\tilde{\alpha}$.

⁷The set of baseline parameters used to illustrate the case without emission trading is: $\kappa_1 = \kappa_2 = 2$, $\kappa_3 = 1$, $\gamma_1 = 1$ and $\gamma_2 = \gamma_3 = 0.5$ (the values satisfy assumption (10) and (11)).

$[\underline{\alpha}, \bar{\alpha}]$ with $\underline{\alpha} = \kappa_1 - \kappa_3$. Considering again that $\omega^R = \alpha$ and following the same approach as in section 4.1, if $\bar{\omega}^R$ is the selected normalized equilibrium then:

$$\sum_{i \neq 2} (\kappa_i - \kappa_2 + (2 + 3\gamma_2 - 3\gamma_i)\alpha - 4w_i^R - 2w_{-i \setminus 2}^R)(w_i^R - w_i) \geq 0 \quad (23)$$

for all \bar{w} such that $w_i + w_{-i} = \alpha$ and $w_i, w_{-i} \geq 0$.

Suppose equation $\kappa_i - \kappa_2 + (2 + 3\gamma_2 - 3\gamma_i)\alpha - 4w_i^R - 2w_{-i \setminus 2}^R = 0$ holds for $i = 1, 3$. Then using the feature that $\omega_2^R = \alpha - \omega_{-2}^R$, one gets

$$\omega_i^R(\alpha) = \frac{2\kappa_i - \kappa_{-i} + \alpha(2 + 3\gamma_{-i} - 6\gamma_i)}{6} \text{ for } i = 1, 2, 3 \quad (24)$$

Since for any $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, $\omega_i^R(\alpha) \in (0, \alpha)$ for all i , these strategies are those chosen by players at the normalized equilibrium.

On the domain of definition of the GEC, $[\underline{\alpha}, \bar{\alpha}]$, all players produce and create emissions as a by-product. It is straightforward to check that setting a binding cap forces all players to reduce their allowances that is, $\omega_i^R(\alpha) < \omega_i^N$ for all $[\underline{\alpha}, \bar{\alpha}]$ for $i = 1, 2, 3$. As in the first scenario, we also observe that allowances and emissions are continuous at $\bar{\alpha}$. So, these properties hold regardless of whether emission trading is possible. Player 2 chooses the highest allowance, even when the quota is binding. Figure 2 depicts the evolution of allowances and emissions when the GEC is moving in $[\underline{\alpha}, \bar{\alpha}]$.⁸ We can observe that the allowance chosen by player 1 is relatively insensitive to the value of the GEC and very close to the allowance strategy at the Nash equilibrium. For low concern players, the allowance strongly changes in response to change in α . In addition, the lower α , the larger the difference between Nash and normalized equilibrium allowances.

5 International control of emissions: what is the best regime?

As far as the international control of pollution is concerned, four different situations can be discussed within our simple framework. These situations differ with respect to first the scenario – with or without emission trading – and second, the regime – with or without a binding GEC. Our aim, in this section, is to examine the different combination of scenarios and regimes, with particular attention being paid to i/ the impact of the introduction of a binding cap on payoffs in each scenario and ii/ the comparison of the performance of the GEC system alone and the ITC

⁸Figures provided for the case with emission trading and for the comparison between the two scenarios have been done using: $\kappa_1 = \kappa_2 = 1.5$, $\kappa_3 = 1$, $\gamma_1 = 6/7$ and $\gamma_2 = \gamma_3 = 4/7$ (theses values are consistent with the non-negativity conditions).

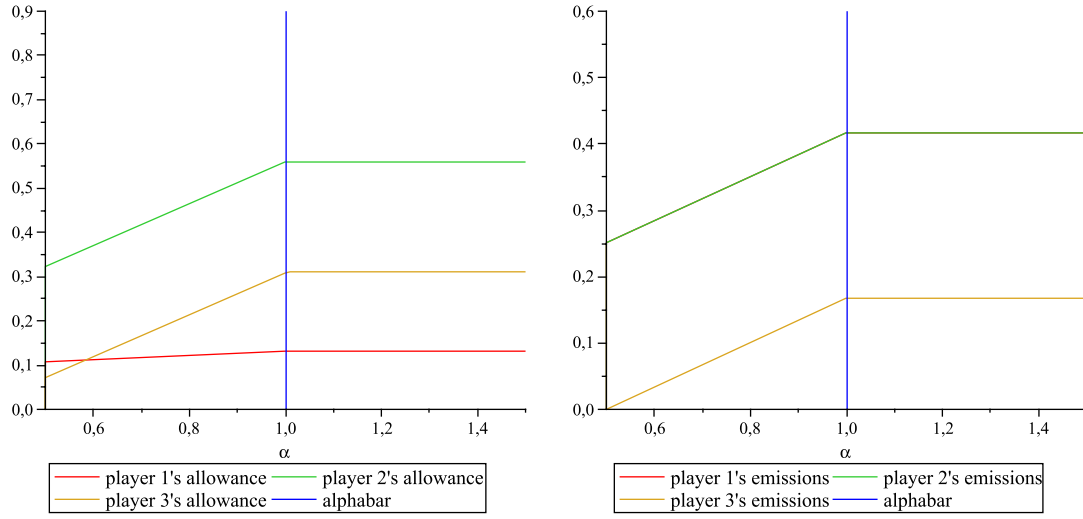


Figure 2: Right: Emission allowances, with and without a binding cap, Left: emission levels

system alone. The GEC system alone has to be understood as the system that consists of the direct choice of emission levels under the coupled constraint (analyzed in section 4.1) whereas the ITC system alone refers to the situation where players choose their emission allowances and are not submitted to a global cap (studied in section 3.2).

Before going any further, recall that part of the task has already been done in section 3.3, where we compared the two usual situations found in the literature: direct choice of emission against choice of emission allowances. This comparison was a means to emphasize the impact of emission trading on both players' strategies and payoffs. This analysis was conducted in the first regime without any binding cap. However, it turns out that all the results we obtained for Nash equilibria (see propositions 1 and 2) can be extended to normalized equilibria. For any cap chosen in $[\underline{\alpha}, \bar{\alpha}]$ ⁹ the high concern - high productivity player is still the one who buys emission permits and chooses an emission allowance lower than the level of emission he would have chosen in the absence of trading. By contrast, the low concern players sell permits on the market and increase their emissions, when they are tradable. Regarding the impact of emission trading on payoffs, when players face the coupled constraint, one can also easily verify that for any $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, emission trading decreases country 1's payoffs and increases the payoffs of country 2 and 3, the overall impact being again positive. Therefore, it appears that the introduction of a binding cap

⁹This is the relevant interval for the comparison between normalized equilibria with and without emission trading because both solutions exist for any α picked up in this interval. In addition, $\underline{\alpha} > \bar{\alpha}$ which implies that all players release positive emissions in both scenarios.

does not affect the analysis of the effect of trading, as discussed in section 3.3.

In the following section, we address the question of how choosing the global emission cap, corresponding to stage 1 of the game.

5.1 Binding vs. non-binding GEC: comparison between payoffs

We do not necessarily seek to determine which cap maximizes a given objective, for instance the unweighted sum of individual payoffs. Rather, we are wondering – in both scenarios – whether setting a binding GEC can be profitable to all players. This is the minimum requirement for an agreement to emerge in stage 1 of the game.

We are looking for the conditions under which no player incurs losses from the introduction of a cap. The analysis relies on a comparison between payoffs attained at the normalized equilibrium, these payoffs being dependent on the endogenous cap, and the payoffs of the Nash equilibrium with a non-binding cap. Expressions of individual payoffs in all scenarios are needed.

Using the property that strategies, and consequently individual payoffs, are continuous at the critical bound $\bar{\alpha}$, $e_i^R(\bar{\alpha}) = e_i^N$ and $\omega_i^R(\bar{\alpha}) = \omega_i^N$, one obtains, for the case where players directly choose their emission level,

$$W_i^R(\alpha) = \frac{1}{36} ((4\kappa_i + \kappa_{-i} - \alpha(2(1 - \gamma_i) + \gamma_{-i}))(2\kappa_i - \kappa_{-i} + \alpha(2(1 - \gamma_i) + \gamma_{-i}))) - \frac{\gamma_i}{2}\alpha^2, \quad (25)$$

$W_i^R(\bar{\alpha}) = W_i^N$ and expressions are symmetric for players $-i$.

In the second scenario with trading, the expression of country i 's payoff (expressions being symmetric for $-i$), with a binding cap is given by

$$V_i^R(\alpha) = \frac{1}{36} ((4\kappa_i + \kappa_{-i} - 2\alpha)(2\kappa_i - \kappa_{-i} + 2\alpha)) + \frac{1}{6}(\kappa_i + \kappa_{-i} - 2\alpha)(\gamma_{-i} - 2\gamma_i)\alpha - \frac{\gamma_i}{2}\alpha^2, \quad (26)$$

with $V_i^R(\bar{\alpha}) = V_i^N$.

Given these expressions, the following result can be established.

Proposition 3 *Regardless of the possibility of trading emission permits, there exists a non empty set of values of α such that imposing a binding emission cap is beneficial to all players.*

Proof. The proof is similar in both scenarios, it uses the features of the value function in the neighborhood of the critical bound $\bar{\alpha}$.

i/ For the sake of simplicity, in the first scenario without trading, attention is paid to the interval $[\tilde{\alpha}^w, \bar{\alpha}]$. Properties of $W_i^R(\alpha)$: $(W_i^R)'(\alpha) \geq 0 \Leftrightarrow \alpha \leq \alpha_i^W$ with $\alpha_i^W = \frac{(2(1-\gamma_i)+\gamma_{-i})(\kappa_i+\kappa_{-i})}{(2(1-\gamma_i)+\gamma_{-i})^2+18\gamma_i}$. In addition, $\alpha_i^W < \bar{\alpha} \Leftrightarrow -\gamma_{-i} + 2\gamma_i + 4 > 0$, which holds under assumptions (8) and (11) for all

i. The function $W_i^R(\alpha)$ is decreasing in the neighborhood of $\bar{\alpha}$ and, since it is continuous at $\bar{\alpha}$, $\exists I_i \subseteq [\tilde{\alpha}^w, \bar{\alpha}]$ such that $\forall \alpha \in I_i$, $W_i^R(\alpha) \geq W_i^N$. This feature holds for $i = 1, 2, 3$. Finally notice that $\cap I_i \neq \emptyset$.

ii/ From the properties of $V_i^R(\alpha)$, we obtain $(V_i^R)'(\alpha) \geq 0 \Leftrightarrow \alpha \leq \alpha_i^V$ with $\alpha_i^V = \frac{(2+3(\gamma_{-i}-2\gamma_i))(\kappa_i+\kappa_{-i})}{2(2+3(2\gamma_{-i}-\gamma_i))}$ and $\alpha_i^V < \bar{\alpha} (\Leftrightarrow 4 + 3(2\gamma_i - \gamma_{-i}) > 0)$ under (8) and (11). The function $V_i^R(\alpha)$ is decreasing in the neighborhood of $\bar{\alpha}$ and continuous at $\bar{\alpha}$, thus there exists an interval $J_i \subseteq [\underline{\alpha}, \bar{\alpha}]$ such that $\forall \alpha \in J_i$, $V_i^R(\alpha) \geq V_i^N$, for all i . Finally one has $\cap J_i \neq \emptyset$, which completes the proof. ■

Therefore, it is possible to choose the cap in such a way that the three players, regardless of their particular environmental concern and/or technology, find it worthwhile to be submitted to the coupled constraint. In other words, controlling world emissions thanks to the emission cap, provided that it is appropriately chosen, is better than letting countries directly choose their emission level provided that they behave strategically. Figure 3 depicts the evolution of players' payoffs as a function of the cap. In this particular example, we even see that player 1 and 3 are always better off under the binding cap regime whereas the cap has to be high enough in order for player 2 to gain from its introduction.

The same result holds when players now non-cooperatively choose emission allowances and their representative firms are allowed to trade permits with each other. In some cases, like the one illustrated in figure 4, all players gain from the introduction of a binding GEC, whatever the level at which it is fixed.

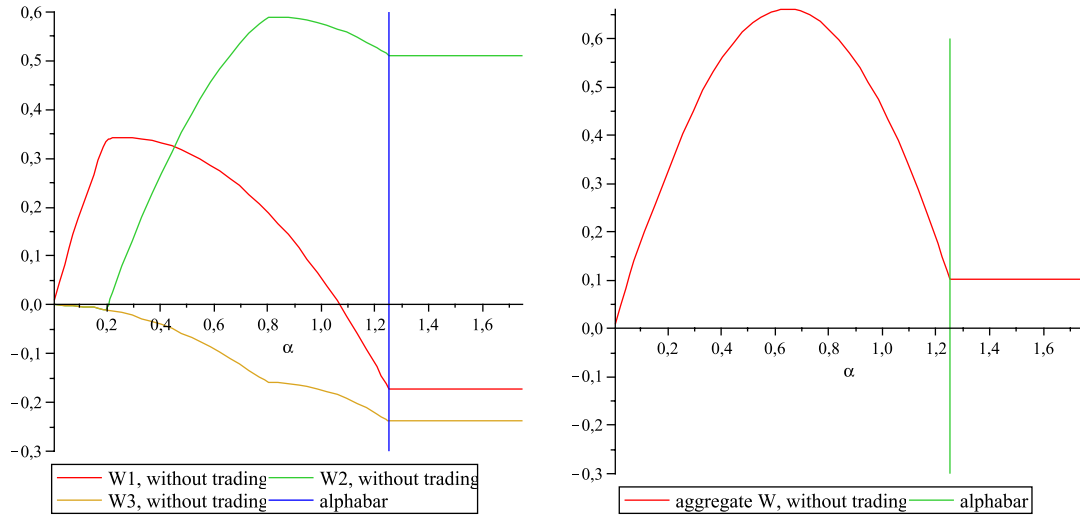


Figure 3: Individuals and aggregate payoffs: binding vs. non-binding

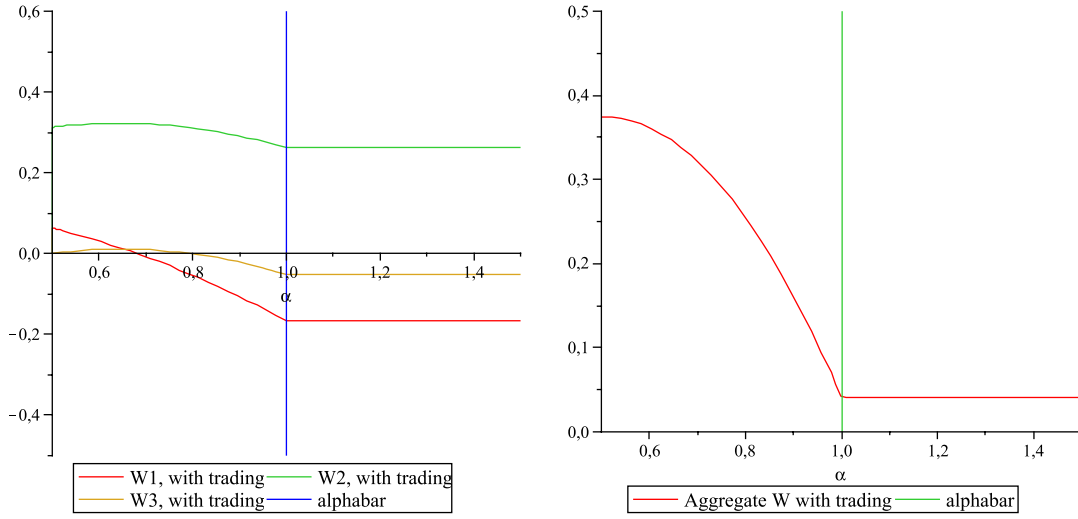


Figure 4: Individuals and aggregate payoffs: binding vs. non-binding

5.2 GEC vs. ITC: what is the best regime?

We now address the central question raised by the present analysis: Is it better to control GHG emissions thanks to a system of individual emission quotas and emission trading or to set a global emission cap and let countries decide their emission level?

We already know that, for any α , the high concern - high productivity player earns the highest payoff in the first scenario without emission trading. In addition, the last proposition states that there exists some values of the GEC for which, in the first scenario, setting a binding cap increases his payoff. Thus, it is clear that one can find values of α such that this player will prefer the GEC system alone rather than the ITC system alone. The comparison is not so easy for players 2 and 3 because, in contrast with player 1, they find it profitable to be allowed to trade emission permits.

To answer this question, let us compare, for these two players, the payoff obtained at the Nash equilibrium, with emission trading, $V_i^N = V_i^R(\bar{\alpha})$, with the payoff at the normalized equilibrium, in the absence of trading, $W_i^R(\alpha)$. The following result can be stated.

Proposition 4 *Suppose*

$$\frac{9\gamma_i^2(\gamma_{-i} - 2\gamma_i - 4)^2}{(2 + \gamma_{-i} - 2\gamma_i)^2 + 18\gamma_i} > (\gamma_{-i} - 2\gamma_i)(5\gamma_{-i} + 2\gamma_i) \quad (27)$$

Then, there exists a subset of $[\underline{\alpha}, \bar{\alpha}]$ such that individual payoffs under the global emission cap regime are higher than the payoffs obtained in the international cap and trade system.

Proof. Let us first compute the difference between the maximum possible payoff under the GEC alone, $W_i^R(\alpha_i^W)$, and the payoff at the Nash equilibrium of the first scenario, $W_i^R(\bar{\alpha})$

$$W_i^R(\alpha_i^W) - W_i^R(\bar{\alpha}) = \frac{9\gamma_i^2(\gamma_{-i} - 2\gamma_i - 4)^2}{36((2 + \gamma_{-i} - 2\gamma_i)^2 + 18\gamma_i)}\bar{\alpha}^2 > 0 \quad (28)$$

Next, compute the difference between the Nash equilibrium payoffs, in the two scenarios

$$V_i^R(\bar{\alpha}) - W_i^R(\bar{\alpha}) = \frac{(\gamma_{-i} - 2\gamma_i)(5\gamma_{-i} + 2\gamma_i)}{36}\bar{\alpha}^2 > 0 \quad (29)$$

To complete the proof, it is sufficient to impose $W_i^R(\alpha_i^W) > V_i^R(\bar{\alpha})$ which is equivalent to (27). This implies that there exist values of α such that players 2 and 3 are better off under the GEC system alone. ■

We are now able to summarize our findings. Consider this opportunity to move away from the business-as-usual scenario where countries directly and non-cooperatively choose their emission levels. Then, two different ways can be followed. On the one hand, the case usually considered in the literature is the one where countries may adopt a cap and trade system. In our setting, allowing emission trading has no impact on the amount of aggregate emissions released in the atmosphere. We also show that if the ITC is accompanied by higher aggregate payoff, one player is always worse off with emission trading. Thus, he will logically not approved any agreement that would crown the trading system. On the other hand, the opportunity exists to set a global emission ceiling, in the case where countries directly choose their emission levels. This is obviously a means to reduce the amount of aggregate emissions, with respect to the business-as-usual. The comparison between the two regimes also reveals that for some values of the cap, all countries gain from the introduction of a binding cap. Finally, our last result provides strong support for the GEC system alone because we show that some values of the GEC exist for which it outperforms the ITC system alone not only in terms of emission reduction but also in terms of individual payoffs. The GEC system alone can thus be unanimously approved when the ITC system cannot. Our numerical example illustrates the existence of a non empty set for the GEC such that for any α picked up in this set, all players are better off under the GEC system alone (see figure 5).

6 Conclusion

We model climate change negotiations as a two-stage game. In the first stage, players have to agree on a global emission cap (GEC). In the second stage, they non cooperatively choose either

their emission level or their emission quota, depending on whether emission trading is allowed, under the cap that potentially binds them together. When the cap is non-binding, there exists a unique Nash equilibrium. When the emission cap is binding, we select a normalized equilibrium by solving a variational inequality with a unique solution. In both cases (with or without emission trading), we show that there exists a non-empty range of values for which setting a binding cap improves all players' payoff. We also show that for some values of the cap, all players get a higher payoff under the GEC system alone than under the international cap and trade (ITC) system alone whereas overall emissions are reduced. Thus, the introduction of a GEC outperforms the ITC system both in terms of emission reduction and of payoff gains.

From a policy perspective, the GEC system, by opposition to the ITC system, presents the advantage of circumventing the critical question of how the initial allocation of permits between countries should be determined. However, this system raises another difficulty regarding how to choose the weights, often interpreted as negotiation powers, attributed to the players in Rosen's formalism. In any event, the following argument provides strong support for the introduction of a GEC. As mentioned by Helm (2003), in general the ITC system cannot at the same time lead to less pollution and be unanimously approved by all the countries involved in the negotiations. The GEC system does not suffer from this weakness because both goals can be simultaneously met by appropriately choosing the cap.

An incoming development of this work consists in extending the analysis to the case of discontinuous damages. Indeed, there is a growing evidence of the existence of thresholds levels of GHG concentrations beyond which catastrophic and irreversible events may occur. Within our static framework, this would induce us to consider critical emission levels at which damages jump upward. These levels should be different among players since countries are not equally exposed to the consequences of global warming. Another natural extension of this analysis is to account for the intertemporal dimension of the climate change problem that is due to the stock pollutant nature of GHG. This can be done by assessing the issue raised by the present paper in a differential game.

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Appendix

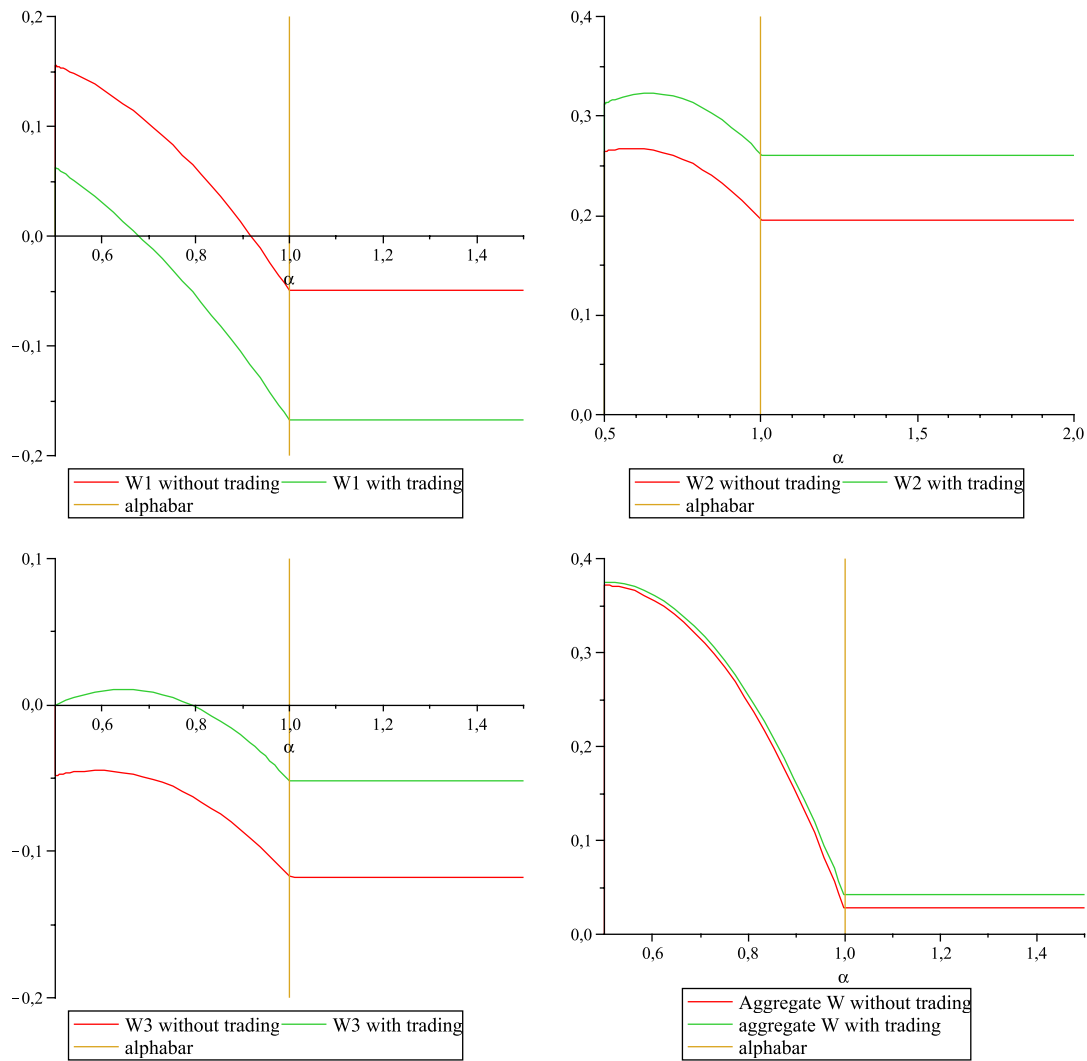


Figure 5: Comparison between payoffs: International cap and trade vs. global emission cap

A Existence of a normalized equilibrium

$\bar{e}^R(\alpha) = (e_1^R(\alpha), \dots, e_n^R(\alpha)) \in \mathcal{E}(\alpha)$ is a selected normalized equilibrium for the pseudogame $\Gamma(\alpha)$ if and only if:

$$\sum_{i=1}^n W_i(\bar{e}^R(\alpha)) = \text{Max}_{\bar{e} \in \mathcal{E}(\alpha)} \sum_{i=1}^n W_i(e_i, \bar{e}_{-i}^R(\alpha)) \quad (30)$$

that is, if and only if: $\bar{e}^R(\alpha)$ is a fixed point of the set-valued function L_α defined on $\mathcal{E}(\alpha)$ by:

$$\bar{u} = (u_1, \dots, u_n) \in \mathcal{E}(\alpha) \longrightarrow L_\alpha(\bar{u}) = \text{ArgMax}_{\bar{e} \in \mathcal{E}(\alpha)} \sum_{i=1}^n W_i(e_i, \bar{u}_{-i}). \quad (31)$$

For $\alpha > 0$, $\mathcal{E}(\alpha)$ and L_α satisfy the assumptions of Kakutani's theorem which guarantees the existence of such a fixed point.

In fact:

1. $\mathcal{E}(\alpha)$ is a nonempty, convex and compact subset of \mathbf{R}^n ;
 2. L_α has closed graph;
 3. $L_\alpha(\bar{u})$ is a nonempty and convex subset of $\mathcal{E}(\alpha)$ for all $\bar{u} \in \mathcal{E}(\alpha)$.
1. The first point is obvious.
 2. Let us show that, for all $\alpha > 0$, L_α has a closed graph over $\mathcal{E}(\alpha)$.

Let $(u_{1,k}, \dots, u_{n,k})_{k \in \mathbf{N}}$ be a sequence such that $\bar{u}_k = (u_{1,k}, \dots, u_{n,k}) \in \mathcal{E}(\alpha)$ for all $k \in \mathbf{N}$ and such that \bar{u}_k converges to \bar{u} in $\mathcal{E}(\alpha)$ as $k \rightarrow \infty$. Then, for all $i \in N$, $u_{i,k}$ converges to u_i as $k \rightarrow \infty$ and, since $\mathcal{E}(\alpha)$ is a closed set, $\bar{u} = (u_1, \dots, u_n) \in \mathcal{E}(\alpha)$.

Moreover, let $(e_{1,k}^R, \dots, e_{n,k}^R)_{k \in \mathbf{N}}$ be a sequence such that:

$$\bar{e}_k^R = (e_{1,k}^R, \dots, e_{n,k}^R) \text{ converges to } \bar{e}^R = (e_1^R, \dots, e_n^R) \text{ as } k \rightarrow \infty \text{ and} \quad (32)$$

$$\bar{e}_k^R \in L_\alpha(\bar{u}_k), \text{ for all } k \in \mathbf{N}. \quad (33)$$

Then, for all $i \in N$, $e_{i,k}^R$ converges to e_i^R as $k \rightarrow \infty$.

One has to prove that $(e_1^R, \dots, e_n^R) \in L_\alpha(u_1, \dots, u_n)$, that is:

$$\bar{e}^R = (e_1^R, \dots, e_n^R) \in \mathcal{E}(\alpha) \text{ and} \quad (34)$$

$$\bar{e} = (e_1, \dots, e_n) \in \mathcal{E}(\alpha) \implies \sum_{i=1}^n W_i(e_i^R, \bar{u}_{-i}) \geq \sum_{i=1}^n W_i(e_i, \bar{u}_{-i}) \quad (35)$$

Due to (32) and (33), we obtain condition (34) using a limit process.

Regarding implication (35), let $\bar{e} = (e_1, \dots, e_n) \in \mathcal{E}(\alpha)$. From (33), one has:

$$\sum_{i=1}^n W_i(e_{i,k}^R, \bar{u}_{-i,k}) \geq \sum_{i=1}^n W_i(e_i, \bar{u}_{-i,k})$$

The function W_i is continuous on \mathbf{R}_+^n , for all $i \in N$, therefore

$$\sum_{i=1}^n \lim_{k \rightarrow \infty} W_i(e_{i,k}^R, \bar{u}_{-i,k}) \geq \sum_{i=1}^n \lim_{k \rightarrow \infty} W_i(e_i, \bar{u}_{-i,k})$$

This implies

$$\sum_{i=1}^n W_i(e_i^R, \bar{u}_{-i}) \geq \sum_{i=1}^n W_i^R(e_i, \bar{u}_{-i}) \quad (36)$$

This is true for all $\bar{e} = (e_1, \dots, e_n) \in \mathcal{E}(\alpha)$. Therefore L_α has closed graph over $\mathcal{E}(\alpha)$.

3. For all $\bar{u} \in \mathcal{E}(\alpha)$, $L_\alpha(\bar{u})$ is a nonempty set since $\mathcal{E}(\alpha)$ is compact and, for all $i \in N$, the function W_i is continuous \mathbf{R}_+^n .

For all $\bar{u} \in \mathcal{E}(\alpha)$, $L_\alpha(\bar{u})$ is a convex set since the function defined by $F_{\bar{u}}(\bar{e}) = \sum_{i=1}^n W_i(e_i, \bar{u}_{-i})$ is assumed to be quasiconcave.

B Continuity with respect to α of the normalized equilibrium and the aggregate value at the equilibrium

Let $\alpha > 0$ and $\alpha_k \in \mathbf{R}_+$ such that $\alpha_k \rightarrow \alpha$ as $k \rightarrow \infty$. We have to show that $\bar{e}^R(\alpha_k) \rightarrow \bar{e}^R(\alpha)$ as $k \rightarrow \infty$.

- There exists a compact set $K_\alpha \subset \mathbf{R}_+^n$ such that $\mathcal{E}(\alpha_k) \subset K_\alpha$ for all $k \in \mathbf{N}$ sufficiently large. So there exists a subsequence $(\bar{e}^R(\alpha_{k_j}))_{j \in \mathbf{N}}$ and a vector $\bar{e}^R \in K_\alpha$ such that: $\bar{e}^R(\alpha_{k_j}) \rightarrow \bar{e}^R$. Since $\bar{e}^R(\alpha_{k_j}) \in \mathcal{E}(\alpha_{k_j})$, for all k_j , and $\alpha_{k_j} \rightarrow \alpha$, we have: $\bar{e}^R \in \mathcal{E}(\alpha)$.
- Now, let $\bar{e} \in \mathcal{E}(\alpha)$. Then, for all $j \in \mathbf{N}$, there exists $\bar{e}_j = (e_{1,j}, \dots, e_{n,j}) \in \mathcal{E}(\alpha_{k_j})$ such that $\bar{e}_j \rightarrow \bar{e}$ as $j \rightarrow \infty$. But $\bar{e}^R(\alpha_{k_j})$ satisfies:

$$\sum_{i=1}^n W_i(\bar{e}^R(\alpha_{k_j})) \geq \sum_{i=1}^n W_i(f_i, \bar{e}_{-i}^R(\alpha_{k_j})) \text{ for all } \bar{f} = (f_1, \dots, f_n) \in \mathcal{E}(\alpha_{k_j})$$

Then:

$$\sum_{i=1}^n W_i(\bar{e}^R(\alpha_{k_j})) \geq \sum_{i=1}^n W_i(e_{i,j}, \bar{e}_{-i}^R(\alpha_{k_j})).$$

Passing to the limit for $j \rightarrow \infty$, we obtain:

$$\sum_{i=1}^n W_i(\bar{e}^R) \geq \sum_{i=1}^n W_i(e_i, \bar{e}_{-i}^R).$$

But this is true for all $\bar{e} \in \mathcal{E}(\alpha)$ then \bar{e}^R coincides with $\bar{e}^R(\alpha)$, the unique normalized equilibrium and we have:

$$\bar{e}^R(\alpha_{k_j}) \rightarrow \bar{e}^R(\alpha) \text{ as } j \rightarrow \infty.$$

- This is true for all convergent subsequences of $\bar{e}^R(\alpha_k)$ so one can prove that this is true for the sequence $(\bar{e}^R(\alpha_k))_{k \in N}$ and at the end we obtain:

$$\bar{e}^R(\alpha_k) \rightarrow \bar{e}^R(\alpha) \text{ as } k \rightarrow \infty, \text{ for all } \alpha_k \rightarrow \alpha$$

which completes the proof.